

Exam 2

March 6, 2014

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Record your answers to the multiple choice problems on this page. Place an \times through your answer to each problem.

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May the odds be ever in your favor!

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Multiple Choice

1. (5 pts.) Let $S = \{A, B, C, D\}$ be the sample space of an experiment with $P(A) = 0.32$, $P(C) = 0.21$ and $P(D) = 0.1$. Find $P(B)$.

- (a) 0.37 (b) 0.46 (c) 0.47
(d) 0.36 (e) The number cannot be determined from the given data.

Sum of probabilities should be 1

2. (5 pts.) If $S = \{s_1, s_2, \dots, s_n\}$ is the sample space of an experiment, which of the following is NOT always true?

- (a) for any s_i , $0 \leq P(s_i) \leq 1$ (b) $P(S) = 1$
 (c) $P(s_1) = P(s_2) = P(s_3) = \dots = P(s_n)$ (d) $P(S) = P(s_1) + P(s_2) + \dots + P(s_n)$
(e) If $E = \{s_1, s_3, s_n\}$ then $P(E) = P(s_1) + P(s_2) + P(s_n)$

c) is only true if all outcomes
equally likely

3. (5 pts.) A French class has 10 students, consisting of 8 women and 2 men. The professor plans to randomly choose a group of 3 students to go on a trip to Quebec. What is the probability that the group includes at least one man?

(a) $1/15$

(b) $7/15$

(c) $2/3$

~~(d) $8/15$~~

(e) $3/5$

$$P(0 \text{ men}) = \frac{C(8,3)}{C(10,3)} = \frac{7}{15}$$

$$P(\geq 1 \text{ man}) = 1 - \frac{7}{15}$$

4. (5 pts.) Moe is given a 4-sided die with the numbers 1 to 4 on it. Larry is given a 6-sided die with the numbers 1 to 6 on it. Curly is given an 8-sided die with the numbers 1 to 8 on it. All of them are told to roll their dice once, which they do so, independently of each other. What is the probability that none of them rolls a 1? (All of the dice are fair)

(a) $\frac{104}{192}$

(b) $\frac{87}{192}$

(c) $\frac{191}{192}$

(d) $\frac{1}{192}$

~~(e) $\frac{105}{192}$~~

$$\begin{array}{ccc} \frac{3}{4} & \times & \frac{5}{6} & \times & \frac{7}{8} \\ \uparrow & & \uparrow & & \uparrow \\ \text{Moe} & & \text{Larry} & & \text{Curly} \end{array}$$

5. (5 pts.) Suppose we have the following information:

$$Pr(E) = \frac{1}{2} \quad Pr(F) = \frac{1}{3} \quad Pr(G) = \frac{1}{4}$$

$$Pr(E \cap F) = \frac{1}{6} \quad Pr(E \cap G) = \frac{1}{8} \quad Pr(F \cap G) = \frac{1}{10}$$

Which pair(s) of events are independent?

- (a) E and G are independent, and F and G are independent.
 (b) E and F are independent, E and G are independent, and F and G are independent.
 (c) E and F are independent, and E and G are independent.
 (d) No two are independent.
 (e) E and F are independent, and F and G are independent.

$$P(E \cap F) = P(E)P(F)$$

$$P(E \cap G) = P(E)P(G)$$

$$P(F \cap G) \neq P(F)P(G)$$

6. (5 pts.) A smartphone security test was conducted in which the phones were held and released to the ground. According to the results, the probability that the screen of the phone will crack is .35. The probability that the battery of the phone will be damaged is .10. The probability that both the screen cracks and the battery is damaged is .055. A student drops his smartphone; given that the screen cracked, what is the probability that the battery will be damaged (all options are rounded to 3 decimal places)?

- (a) 0.250 (b) 0.004 (c) 0.157 (d) 0.100 (e) 0.500

$$P(B|S) = \frac{P(B \cap S)}{P(S)} = \frac{.055}{.35}$$

\uparrow battery damaged \uparrow screen cracked

7. (5 pts.) A box contains five light bulbs, of which three are defective and two are good. Bob takes one out at a time and tests it, and he stops as soon as he has found a good one. At no point does he put any bulbs back in the box. What is the probability that he finds the first good one on the third try?

(a) $\frac{13}{20}$

(b) $\frac{1}{10}$

(c) $\frac{3}{10}$

~~(d) $\frac{1}{5}$~~

(e) $\frac{2}{5}$

$$\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$$

\uparrow $P(\text{1st bad})$ \uparrow $P(\text{2nd bad} | \text{1st bad})$ \uparrow $P(\text{3rd good} | \text{1st, 2nd bad})$

8. (5 pts.) Jennifer and Katie work on a mathematics problem independently. The probability that Jennifer solves the problem is 0.78 and the probability that Katie solves the problem is .67. What is the probability that exactly one of them solves the problem?

(a) 0.5226

~~(b) 0.4048~~

(c) 0.2574

(d) 0.1474

(e) 0.4774

$$\left. \begin{aligned} P(J \text{ only}) &= .78 \times .33 \\ P(K \text{ only}) &= .67 \times .22 \end{aligned} \right\} \text{add these}$$

9. (5 pts.) A coin is weighted so that it comes up heads five eighths of the time, and comes up tails three eighths of the time. The coin is tossed three times. What is the probability of getting at least 2 tails?

- (a) ~~162/512~~ (b) 45/512 (c) 225/512
 (d) 135/512 (e) 350/512

$$P(\text{exactly 2 tails}) = C(3, 2) \left(\frac{3}{8}\right)^2 \left(\frac{5}{8}\right) = \frac{135}{512}$$

$$P(\text{exactly 3 tails}) = \left(\frac{3}{8}\right)^3 = \frac{27}{512}$$

} add these

10. (5 pts.) A restaurant owner decided to keep track of the number of vegetable orders made in a given evening. He produced the following frequency distribution:

Vegetable	Frequency
broccoli	15
cauliflower	10
asparagus	12
peas	13
green beans	20

If instead the restaurateur decided to produce a relative frequency table, what number would appear to the right of "green beans"?

- (a) 1/5 (b) 1/4 (c) 1/7
 (d) ~~2/7~~ (e) The number cannot be determined from the given data.

$$\frac{20}{15 + 10 + 12 + 13 + 20} = \frac{2}{7}$$

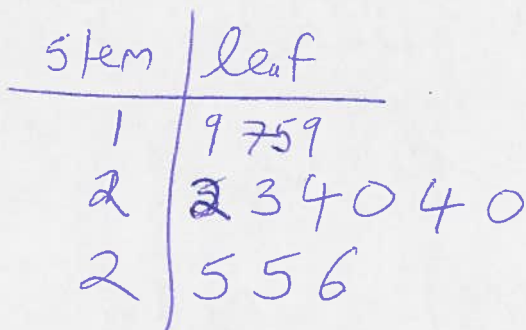
Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The percentage of active ingredient on 13 pills was measured. The results are:

~~19, 25, 22, 25, 17, 15, 23, 24, 20, 24, 26, 19, 20.~~

(a) Draw a stem-and-leaf plot using the intervals 15-19, 20-24, 25-29.

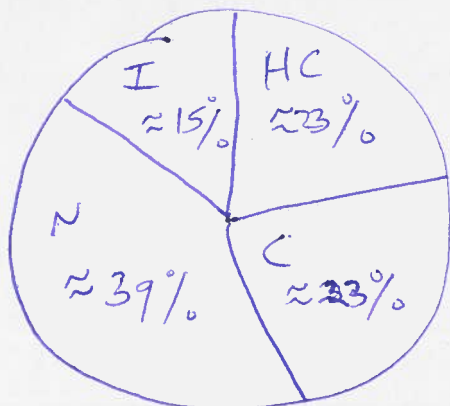


(b) The company considers a pill to be highly concentrated (HC) if the percentage of active ingredient is between 25-29, concentrated (C) if it is between 23-24, neutral (N) if it is between 18-22, and ineffective (I) if it is between 15-17. Draw a relative frequency histogram below, using the categories HC, C, N, and I.



HC $\frac{3}{13}$
 C $\frac{3}{13}$
 N $\frac{5}{13}$
 I $\frac{2}{13}$

(c) Draw a pie chart that states the percentage of pills that are HC, C, N and I.



12. (10 pts.) Nine busses stop at my local mall. Three — the L7, L11 and L20 — are local, and six — the X3, X8, X13, X15, X17 and X18 — are express. The L20, X3 and X8 stop at the south end of the mall, and the rest stop at the north end. At a particular moment, the next bus to arrive at the mall is equally likely to be any of the nine. I am interested in three events:

- A : the next bus is local
- B : the next bus is express
- C : the next bus stops at the south end of the mall.

(a) Compute the probabilities of A , B and C .

$$P(A) = \frac{3}{9} \quad P(B) = \frac{6}{9} \quad P(C) = \frac{3}{9}$$

(b) Are the events A and B independent?

$$P(A \cap B) = 0 \neq P(A)P(B) \text{ so } \underline{\text{No}}$$

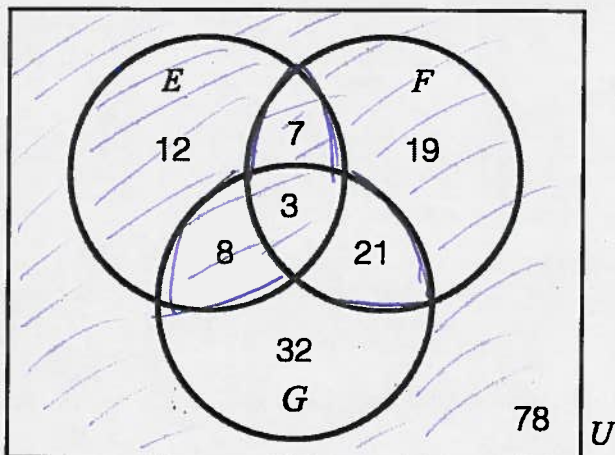
(c) Are the events A and C independent?

$$P(A \cap C) = \frac{1}{9} = P(A)P(C) \text{ so } \underline{\text{YES}}$$

(d) Write down a pair of events (from among A , B and C) that are mutually exclusive.

A and B have no outcome in common.

13. (10 pts.) The Venn diagram below shows the number of people, in a class of 180 students, that belong to the band (event E), play intramural dodgeball (event F) and play handbells (event G). A student is chosen at random from the class.



(a) Compute $P(E|F)$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{(7+3)/180}{(7+19+3+21)/180} = \frac{10}{50}$$

(b) Compute $P(E \cup G')$

$E \cup G'$ is shaded area above

$$P(E \cup G') = \frac{12+7+8+3+19+78}{180} = \frac{127}{180}$$

(c) Given that the randomly chosen student is involved in at least one of the activities (band, dodgeball, handbells), what is the probability that he/she is involved in at least two of them?

Sample of students involved in at least one activity has size $180 - 78 = 102$

Of those, $7+3+8+21 = 39$ are involved in at least two. So prob = $\frac{39}{102}$

14. (10 pts.) Five runners, Adam, Beth, Charlie, Dana and Eric, take part in a running race. The runners are equally matched, so all finishing orders are equally likely. There are no ties. Prizes are given to the first two finishers.

(a) How many possible finishing orders are there?

$$5! = 120$$

(b) What is the probability that neither Adam nor Beth get a prize (recall, prizes are given to the first two finishers)?

ways for A and B not to finish in top 2:

$$3 \times 2 \times 3 \times 2 \times 1 = 36$$

\uparrow choices for first \uparrow 2nd \uparrow 3rd \uparrow 4th \uparrow 5th

So prob = $\frac{36}{120}$

(c) Suppose we know that Charlie won the race (that is Charlie finished first). Given that extra information, what is the probability that neither Adam nor Beth get a prize?

ways for C to win, A and B not to get

$$\text{prize} : 1 \times 2 \times 3 \times 2 \times 1 = 12$$

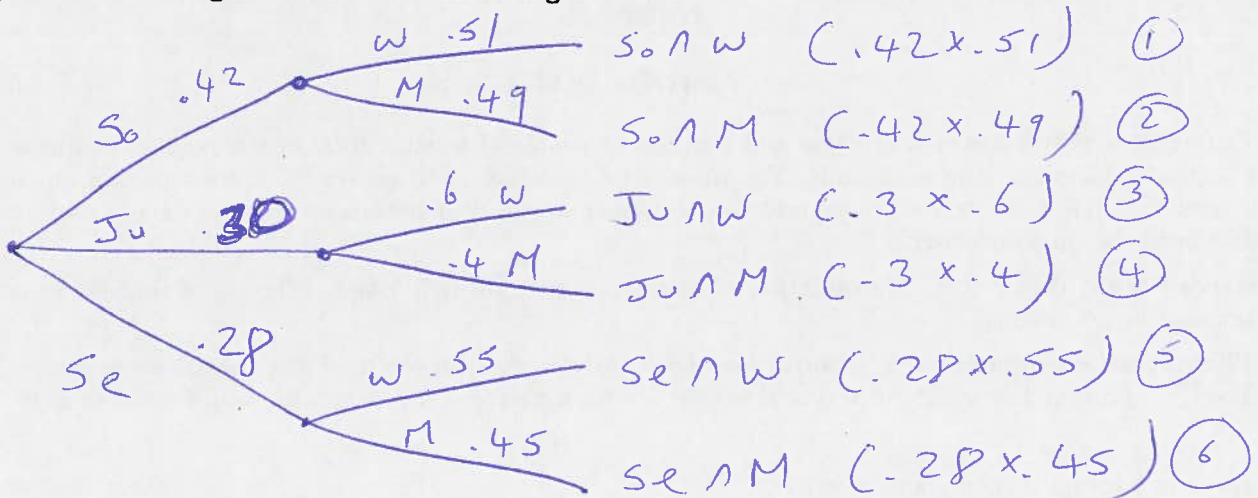
$$\begin{array}{c} \uparrow \\ 1^{\text{st}} \\ \uparrow \\ 2^{\text{nd}} \\ \text{(either D or E)} \end{array}$$

$$\begin{aligned} \# \text{ ways for C to finish first} &: 1 \times 4 \times 3 \times 2 \times 1 \\ &= 24 \end{aligned}$$

$$\text{So prob} = \frac{12}{24} = \frac{1}{2}$$

15. (10 pts.) A poll among college students was conducted and the results are as follow: 42% are sophomores, 30% are juniors and 28% are seniors. Among the sophomores, 51% are women and 49% are men; among the juniors 60% are women and 40% are men; among the seniors 55% are women and 45% are men.

(a) Draw a tree diagram with the information given above.



(b) What is the probability that a student selected at random is either a senior or a man?

Outcomes (2), (4), (5) and (6), so

$$.42 \times .49 + .3 \times .4 + .28 \times .55 + .28 \times .45 = .6058$$

(c) Given that a student selected at random is a man, what is the probability that he is a junior?

$$P(Ju|M) = \frac{P(Ju \cap M)}{P(M)} = \frac{.3 \times .4}{.42 \times .49 + .3 \times .4 + .28 \times .45} = .265 \dots$$

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